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# PROVISIONAL APPLICATION FOR PATENT COVER SHEET

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Additional inventors are being named on the separately numbered sheets attached hereto								
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#### SYSTEM AND METHOD FOR A HYBRID CLOCK AND PROXY AUCTION

#### Field of the Invention

The present invention relates to improving computer-implemented auctions and, more particularly, to computer implementation of a hybrid auction combining elements of a clock auction and a proxy auction.

## Background of the Invention

Sellers of large or complex assets need to consider how to divide and package the assets. Packaging decisions are potentially important whenever there are value dependencies among items in the sense that a bidder's value for a package is different from the sum of the values of the separate parts. For example, a wireless telephone company purchasing radio spectrum rights may realize synergies from obtaining geographically-adjacent licenses. A flower wholesaler in Holland may incur fixed costs for shipping, handling and overhead that make single lot transactions unprofitable. Since flowers are highly perishable, it may also want to limit its purchases to what it can quickly resell. In real estate sales, some potential buyers may be interested in a whole complex of properties while others simply want space for individual homes or businesses. All of these are examples of value dependencies and all can make buyers interested in the way items are packaged for sale.

In practice, sellers accommodate these packaging preferences in a variety of ways. For example, government-run spectrum auctions are invariably preceded by political processes in which potential buyers press their cases about such matters as the allowed uses of the spectrum and the scope of the licenses in terms of bandwidth, band composition, and geographic coverage. In private auctions of relatively homogeneous goods, winning bidders may be allowed to purchase as many similar lots as they like at the winning price before bids are taken for the remaining lots. In the real estate example, bids might be taken both for a whole complex and for its individual properties, and the two constellations of prices compared.

In the last few years, there has been growing interest in auction processes that allow bidders much greater freedom to name the packages on which they bid during the auction. Processes like that described for the real estate example, which determine the packaging, pricing and allocation decisions, can be called "package auctions" or "auctions with package bidding." Typically, bidders in these auctions describe the packages that they wish to acquire and make bids for the named packages.

One especially promising version of a package auction is a proxy auction. It is effectively a sealed-bid auction. The items for sale are taken to be M exogenously given "goods", and thus there are  $2^M-1$  possible packages. The bidder inputs value information into a proxy agent, so that the proxy agent possesses information concerning the value of some or all the  $2^M-1$  possible packages. The proxy agent then submits package bids on behalf of the bidder, selecting one or more packages that optimize the difference between the bidder's value and the amount that can be bid for the package. The auctioneer then selects provisionally-winning bids by solving the optimization problem of selecting bids, at most one from each bidder, that optimize revenues subject to a feasibility constraint. Proxy agents for bidders who are not selected as provisional winners then submit new bids, and the process continues until the bidders who are not provisional winners have no profitable bids remaining to be placed. (For a longer discussion, see "System and Method for a Dynamic Auction with Package Bidding," International Patent Application No. US01/43838.)

In the last few years, there has also been growing practical interest in *clock auctions*. These are auction processes in which the auctioneer announces prices, bidders respond with quantities, the prices are adjusted according to the amount of excess demand, and the process is allowed to repeat. In a situation where the auctioneer is selling goods, the process concludes when prices are reached such that the aggregate demand for each item is no greater than the available supply. Such auction processes are particularly effective in allocating multiple units of multiple types of goods. (For a longer discussion, see "System and Method for an Auction of Multiple Types of Items," International Patent Application No. US02/16937.)

The present invention primarily concerns *hybrid auctions* that combine clock auctions with proxy auctions. These are multi-item auctions that include two phases: an earlier phase in which bidders participate in a clock auction (or other dynamic auction); and a later phase in

which bidders participate in a proxy auction (or some other auction that allows package bidding). By combining the earlier phase and the later phase as in some of the embodiments described herein, it is possible to combine the advantages of the dynamic auction and the advantages of the package auction. In particular, if the earlier phase is a clock auction and the later phase is a proxy auction, then the resulting hybrid auction will combine the transparency and simplicity of the clock auction with the efficient outcome and competitive revenues of the proxy auction.

Various systems and methods in the art facilitate the operation of computer-implemented auctions. The implementation of auctions on computers holds numerous advantages over the earlier art. It facilitates the simultaneous auctioning—in a single, combined auction process—of a plurality of items that are related, for example, in the sense that bidders may value the items as substitutes or complements. It permits a dynamic bidding process for such a plurality of items, in which bidders in diverse locations across the continent or the globe are able to actively participate and to receive feedback in real time about their opponents' bids. It enables the practical introduction of auctions with clocks or package bidding. And in accomplishing the above, it encourages bidders to bid aggressively and straightforwardly for the packages they want, incorporating all available information, and resulting in items being allocated to the bidders who value them the most, while also ensuring a competitive price for the seller or sellers.

The present invention is useful for "reverse auctions" conducted by or for buyers to acquire various kinds of items or resources, "standard auctions" conducted by sellers in which items are offered for sale, and "exchanges" in which both buyers and sellers place bids. Although terms such as "items or quantities demanded" (by a bidder) and "demand curve" (of a bidder) are used to describe the present invention, the terms "items or quantities offered" (by a bidder) and "supply curve" (of a bidder) are equally applicable. In some cases, this is made explicit by the use of both terms, or by the use of the terms "items or quantities transacted" (by a bidder) and "transaction curve" (of a bidder). The term "items or quantities transacted" includes both "items or quantities demanded" and "items or quantities offered". The term "bid" includes both offers to sell and offers to buy. The term "transaction curve" includes both "demand curve" and "supply curve". Moreover, any references to "items or quantities being offered" includes both "items or quantities being sold" by the auctioneer, in the case this is a standard auction for selling items, as

well as "items or quantities being bought or procured" by the auctioneer, in the case this is a reverse auction for buying items or procuring items.

Moreover, while standard auctions to sell typically involve ascending prices, the present invention may utilize prices that ascend and/or descend.

Throughout this document, the terms "objects", "items", "units" and "goods" are used essentially interchangeably. The inventive system and method may be used both for tangible objects, such as real or personal property, and intangible items, such as telecommunications licenses or electric power. The inventive system and method may be used in auctions where the auctioneer is a seller, buyer or broker, the bidders are buyers, sellers or brokers, and for auction-like activities which cannot be interpreted as selling or buying. The inventive system and method may be used for items including, but not restricted to, the following: public-sector bonds, bills, notes, stocks, and other securities or derivatives; private-sector bonds, bills, notes, stocks, and other securities or derivatives; communication licenses and spectrum rights; clearing, relocation or other rights concerning encumbrances of spectrum licenses; electric power and other commodity items; rights for terminal, entry, exit or transmission capacities or other rights in gas pipeline systems; airport landing rights; emission allowances and pollution permits; and other goods, services, objects, items or other property, tangible or intangible. It may also be used for option contracts on any of the above. It may be used in initial public offerings, secondary offerings, and in secondary or resale markets.

The network used, if any, can be any system capable of providing the necessary communication to/from a Bidding Information Processor (BIP), a Bidding Terminal (BT), and an Auctioneer's Terminal (AT). The network may be a local or wide area network such as, for example, Ethernet, token ring, the Internet, the World Wide Web, the information superhighway, an intranet or a virtual private network, or alternatively a telephone system, either private or public, a facsimile system, an electronic mail system, or a wireless communications system, or combinations of the foregoing.

## Brief Description of the Drawings

Figure 1 is a graphical depiction of the architecture of an exemplary computer system in accordance with an embodiment of the invention;

Figure 2 is a graphical depiction of another exemplary computer system in accordance with an embodiment of the invention;

Figure 3 is a detail of one element of the computer system of Figure 2;

Figure 4 is a flow diagram of an exemplary hybrid auction in accordance with an embodiment of the invention;

Figure 5a is a graphical depiction of the architecture of an exemplary auction system in which bidding is intermediated by proxy agents, in accordance with an embodiment of the invention;

Figure 5b is a flow diagram of an exemplary proxy auction phase, in accordance with an embodiment of the invention;

Figures 6a and 6b are flow diagrams illustrating, in greater detail, elements of the flow diagram of Figure 5b;

Figures 7a and 7b are flow diagrams illustrating, in greater detail, elements of the flow diagram of Figure 5b;

Figure 8 is a flow diagram of an exemplary clock auction phase, in accordance with one embodiment of the invention;

Figures 9a and 9b are flow diagrams illustrating, in greater detail, elements of the flow diagram of Figure 8; and

Figure 10 is a flow diagram illustrating, in greater detail, an element of the flow diagram of Figure 8.

## **Detailed Description of Preferred Embodiments**

#### Overall Structure of Auction System

Earlier auction methods and systems are described in U.S. Patent Nos. 5,905,975, 6,021,398 and 6,026,383. The following description will detail the flow of the novel features of

the preferred embodiments of the present method and system for a hybrid auction combining a clock auction (or other dynamic auction) with a proxy auction (or other auction allowing package bidding).

Before describing the auction process in detail, reference is made to Figure 1 to describe the architecture of an exemplary computer system in accordance with an embodiment of the present invention. In the graphical depiction of Figure 1, the computer system consists of multiple bidder and auctioneer computers or terminals 20a-n and 30 communicating with the server (or auction computer) 10 over a network 40. The computers or terminals 20a-n are employed by bidders, the computer or terminal 30 is employed by the auctioneer, and the server 10 is the auction computer. The server 10 consists of a CPU 11, memory 12, a data storage device 13, a communications interface 14, a clock 15, an operating system 16, and an auction program 17. In one embodiment, the system architecture is as a client-server system: the auction computer is a server; and the bidder and auctioneer computers are clients.

Figure 2 is another graphical depiction of an exemplary computer system in accordance with an embodiment of the present invention. The auction system of Figure 2 includes an auction computer 60 (sometimes also referred to as a Bidding Information Processor or BIP), a plurality of user systems 70a, 70b and so on (sometimes also referred to as Bidder Terminal or BT), each user system 70a-n representing an individual bidder, and a user system 80 (sometimes also referred to as an Auctioneer Terminal or AT). The systems 60, 70a-n, and 80 communicate over a network 90. The network represents any system capable of providing the necessary communication to/from BIP, BT, and AT. The network may be a local or wide area network such as, for example, Ethernet, token ring, the Internet, the World Wide Web, the information superhighway, an intranet or a virtual private network, or alternatively a telephone system, either private or public, a facsimile system, an electronic mail system, or a wireless communications system. Each of the systems 60, 70a-n, and 80 may include a typical user interface 65, 75a-n, 85 for input/output which may include a conventional keyboard, display, and other input/output devices. Within each of the systems, the user interface (65, 75a-n, 85) is coupled to a network interface (64, 74a-n, 84), which in turn communicates via the network 90. Both the user interface and network interface connect, at each system, to a CPU (62, 72a-n, 82). Each system includes a memory (66, 76a-n, 86). The BIP 60 also includes a clock 61 and a data storage device 63,

which will ordinarily contain a database. (However, in some embodiments the database might instead be stored in memory 66.) The memory 66 of the BIP 60 can further be broken down into a program 67, data 68 and an operating system 69. The memory (76a-n, 86) of the BT's 70a-n and the AT 80 may include a web browser (for example, Internet Explorer or Netscape) (79, 89) or other general-purpose software, but not necessarily any computer program specific to the auction process. In each system the CPU (62, 72a-n, 82) represents a source of intelligence when executing instructions from the memory (66, 76a-n, 86) so that appropriate input/output operations via the user interface and the network interface take place as is conventional in the art. The particular steps used in implementing the inventive auction system and method are described in more detail below. In one embodiment, each of the user systems is a personal computer or workstation.

Figure 3 is a more detailed illustration of an exemplary BIP 60 showing details of the database. As discussed for Fig. 2, the database is ordinarily stored on a data storage device 63, although in some embodiments it might instead be stored in memory 66. As depicted in Fig. 3, the database includes provision for creating, storing, and retrieving records representing Items in the Auction 63-1, Status of the Items in the Auction 63-2, Auction Timetable 63-3, Current Price(s) 63-4, List of Bidder ID's 63-5, List of Passwords 63-6, Bidding History 63-7, and Constraints on Bids 63-8. The particular set of data required for any particular auction and the format of that datum or data (such as scalar, vector, list, etc.) is more particularly specified by the detailed description of that auction.

#### Bidders, Items, and Package Bids

There are n bidders, often subscripted by i (i = 1,...,n), participating in the auction. Typically, we restrict  $n \ge 2$ , so that there are two or more bidders in the auction. Let  $\Omega$  denote any set of items which are offered at auction. The object of the auction is to allocate, among the bidders, each element of the set  $\Omega$ . Often, but not always,  $\Omega$  will comprise a set of <u>dissimilar items</u>, meaning that if A and B are any two items in  $\Omega$ , then A and B are neither identical nor very close substitutes to one another.

In many preferred embodiments of the present invention, bids comprise pairs, (S, P), where  $S \subset \Omega$  is a subset of the set of all items being auctioned and P is a price at which the

bidder is offering to transact for the subset S. Stated differently, a bid comprises a package of items and an associated price for the package. Such a bid comprising a pair, (S, P), is defined to be a package bid.

In the event that some of the elements of the set  $\Omega$  are identical, we may prefer instead to allow bids to comprise a quantity of each "type" of item. This requires using somewhat different notation. Type is defined as follows: within each type, all of the items are identical or very close substitutes; however, between different types, the items are permitted to be dissimilar. There are  $m \ (m \ge 1)$  types of items are being auctioned. If the bidders are superscripted by i, where (i = 1, ..., n), then in the "type" notation, a bid would comprise a quantity vector  $(Q_1^i, ..., Q_m^i)$  of each type of item, and a price, P, at which the bidder is offering to transact for the entire quantity vector. Such a bid  $(Q_1^i, ..., Q_m^i; P)$  is defined to be a package bid in the "type" notation.

#### Flexible Bid Information and Proxy Agents

Flexible bid information is data that a bidder selects for present or future use by entering into a computer (e.g., a bidder computer or a BT), but at least some of such data is stored in a database rather than being directly and immediately submitted as a bid in an auction. Flexible bid information can include a scalar value, a vector value or a function. The flexible bid information may be an expression of which (or how many units of) item(s) a bidder is willing to purchase at a given price(s), how much money a bidder is willing to pay for the purchase of a given item(s), or any other expression of the willingness-to-pay or value which a bidder places on item(s). It may also include an expression of how much money or other consideration a bidder is willing to spend in aggregate for all of the items purchased. Optionally, flexible bid information may include a bidding rule that contains a limitation (e.g., "I desire up to a quantity of x at a price P, but I do not want any positive quantity at all unless I receive a minimum quantity of y"). Thus, flexible bid information may include one or more bidding rules that may comprise unconditional bids or contingent bids, and may include one or more functions from available information to bid quantities (e.g. a function of the previous bid(s) submitted).

Within the specific context of an auction with package bidding, flexible bid information may include valuation information, budget information, and other information. <u>Valuation</u> information comprises data relating one or more subsets of the set of all items to indices of price

or value, often measured in dollars or other monetary units. For example, valuation information in a package auction for the items {A,B,C} may include a measure of the valuation or cost that a bidder attaches to each of the subsets Ø, {A}, {B}, {C}, {A,B}, {A,C}, {B,C} and {A,B,C}. Budget information comprises data relating to an aggregate index of price or payment, often measured in dollars or other monetary units. For example, budget information in a package auction for the items {A,B,C} may include a measure of the overall budget limit or parameter for whatever items that a given bidder may sell or buy. Other information comprises data relating to the auction that is neither valuation information nor budget information.

The state of the auction system refers to the full history of bids and messages submitted by or on behalf of bidders in the auction process, the full history of messages submitted on behalf of the auctioneer, the full history of constraints imposed by the auction system, and any other relevant information about the progress of the auction. In some of the preferred embodiments of the inventive system and method, bidders are permitted to change or are not permitted to change their flexible bid information, according to rules based on the state of the auction system. In that event, the state of auction system may itself include a list of the past time or times at which bidders were allowed to change their flexible bid information, as well as information about the progress of the auction since this time or these times. In some embodiments the state of the auction system is limited to information reaching the auction computer. However, in other embodiments it includes inputs from the bidder representing flexible bid information. The "state of the auction system" is sometimes referred to, more compactly, as the "auction state information."

The <u>current auction information</u> refers to the portion of the state of the auction system that is made available to bidders. In some preferred embodiments, the auction is conducted in discrete rounds, and bidders are provided with full information about previous rounds, so that the current auction information in a given auction round may include the history of bids and messages submitted by or on behalf of bidders in the auction process, up until and including the previous auction round. In other preferred embodiments, the auction is conducted in discrete rounds, but bidders are provided with less than full information about previous rounds, and so the current auction information in a given auction round may include only a very abbreviated summary of the history of bids and messages submitted by or on behalf of bidders in the auction

process, up until and including the previous auction round. In other preferred embodiments, the auction is conducted in continuous time, and the current auction information at a given time may include the history of bids and messages submitted by or on behalf of bidders in the auction process, with some amount of time lag.

A proxy agent is a computer-implemented system which may submit bids or send messages on behalf of a bidder, based on flexible bid information, current auction information, and/or the state of the auction system. Thus, the inputs of the proxy agent may include flexible bid information; and the outputs of the proxy agent may include bids or messages. Another way to describe this is that a proxy agent may take flexible bid information as instructions and may submit bids or send messages on behalf of a bidder. A proxy agent may be a subsystem of a larger computer-implemented auction system, or it may be a stand-alone, computer-implemented system that is capable of interacting with a computer-implemented auction system.

In some embodiments of the inventive system and method, the bidding may be intermediated by proxy agents. More precisely, a bidder may enter flexible bid information at a bidder computer or a BT, and a proxy agent may submit bids on behalf of the bidder: this process will often be referred to as proxy bidding. In such embodiments of the inventive system and method, proxy bidding may either be voluntary or mandatory. One purpose of voluntary proxy bidding is to facilitate participation by bidders in a dynamic auction. With voluntary proxy bidding, a bidder who expects to be busy during part or all of a dynamic auction can instruct a proxy agent to bid in his (or her) place. One purpose of mandatory proxy bidding is to limit the possibilities for collusion among bidders. For example, it may be believed that bidders can tacitly collude by making use of retaliatory strategies: if bidder ABC raises the high bid on an item of interest to bidder XYZ, an example of a retaliatory strategy would be for bidder XYZ to respond by raising the high bid on an item of interest to bidder ABC. With mandatory proxy bidding, the auctioneer may require bidder XYZ to input his (or her) valuation information into a proxy agent that is incapable of carrying out a retaliatory strategy, effectively limiting the possibilities for collusion among bidders.

Furthermore, in an auction system or method with proxy bidding, a bidder may be allowed to make changes to the flexible bid information that is used by its proxy agent, or a bidder may not be allowed to make such changes. Obviously, a restriction on changes to the

flexible bid information has the greatest force in an auction system where proxy bidding is mandatory. Moreover, the setting on an auction system as to whether a bidder is allowed to make changes may itself be changed over time (or status), and may depend on the history of bidding (or on the identity) of the bidder. For example, bidder i may be allowed to change its flexible bid information early in the auction, but the same bidder i may not be allowed to make changes in its flexible bid information beyond a certain time in the auction. The change in setting for bidder i may depend on the course of bidder i's bidding in the auction. For example, the setting that bidder i is not allowed to make further changes to its flexible bid information may be triggered by the fact that bidder i (or its proxy agent) has submitted insufficiently few new bids between time t and time t+1 of the auction.

#### Definitions Concerning "Types" of Items and "Groups" of Types of Items

Many of the most useful embodiments of the present invention apply in situations where an entity wishes to sell or buy multiple "types" of items or commodities. Furthermore, related types of items may be usefully organized as "groups" of items. In order to describe the preferred embodiments, it will be helpful for us to define some terminology.

<u>DEFINITION 1</u>: A type of item comprises a (nonempty) subset of the set of all items being auctioned, such that any two items within the same type are identical items or close substitutes. Meanwhile, types are defined so that any two items of different types exhibit significant differences in time, location or any other product characteristics. Typically, there are multiple units of each type of item.

In what follows, we will assume that  $m \ (m \ge 1)$  types of items are being auctioned, and that there are  $n \ (n \ge 1)$  bidders participating in the auction. Whenever we state that there is a "plurality of types of items," we are restricting attention to the case where  $m \ge 2$ . The items in the auction may also be said to be "homogeneous" in the case where m = 1 and to be "heterogeneous" in the case where  $m \ge 2$ . Whenever we state that there is a "plurality of bidders," we are restricting attention to the case where  $n \ge 2$ .

DEFINITION 2: A group G of types comprises a (nonempty) subset of  $\{1, \ldots, m\}$ , the set of all types of items. In other words, a group is one or more types of items that are usefully

treated together. Typically, the reason that two types of items are included in the same group is that they are related, for example, they may be contracts for provision of the same commodity, covering different but overlapping time periods.

In what follows, we will assume that the items in the auction are organized so that there are  $h(h \ge 1)$  groups of types of items. There is no requirement that each group contains the same number of types of items, but this often happens to be the case. Whenever we need notation indicating the number of types of items contained in the exemplary group G, our notation will be that there are  $g(g \ge 1)$  types of items in group G. In some of the preferred embodiments of the present invention, the prices for the various types of items within the same group are established so as to maintain a relationship according to a schedule. In others of the preferred embodiments of the present invention, a bidder submits a bid that includes the price for one type of item within a group, and a fixed relation among these prices implies the prices of the other types of item within the same group. In others of the preferred embodiments of the present invention, a bidder submits a bid that includes a one-dimensional price parameter, and this price parameter implies prices for all of the types of items within the group based on a fixed relation among the prices of the types of items.

Some examples of "types" and "groups," in situations where there may be significant commercial possibilities for embodiments of the present invention, include the following:

- Treasury bills or other securities: A government or central bank may wish to auction 3-month, 6-month and 12-month Treasury securities (for example, with the same starting date) together. Thus, there is one group of types of items (h = 1), and it contains three types of items (g = 3). In total, there are three types of items (m = 3).
- Electricity contracts: An electric generating company may wish to simultaneously auction some forward contracts or options contracts for base-load and peak-load electricity generation, with durations of 2 months, 3 months, 6 months, 12 months, 24 months and 36 months, respectively. Thus, there are two groups of types of items (h = 2), each containing six types of items (g = 6). In total there are 2 × 6 = 12 types of items (m = 12).

Two unrelated, heterogeneous consumer commodities (e.g., apples and oranges).
 There are two groups of types of items (h = 2), each containing just one type of item (g = 1), for a total of two types of items (m = 2).

An auction in accordance with an embodiment of the present invention proceeds as follows. First, the auctioneer (i.e., the auctioneer terminal) determines a starting price vector,  $(P_1, \ldots, P_m)$ , and transmits it to the bidding information processor, which in turn transmits it to bidders (i.e., bidder terminals). Second, a bidder responds with a bid vector indicating the quantity of each respective type of item that the bidder wishes to transact at the current price vector. Let the bidders be superscripted by i, where  $(i = 1, \ldots, n)$ . The bid vector for bidder i is denoted by  $(Q_1^i, \ldots, Q_m^i)$ . The following definitions are helpful in describing the process associated with a first embodiment of the present invention.

#### OTHER DEFINITIONS.

The <u>available quantities</u> may, in principle, be specified for each *group* of types of items or for each type of *items*. In the text that follows, we will usually specify the available quantity for the group. The available quantity for group G will be denoted  $\overline{Q}^G$ , and this refers, in the case of an auction to sell, to the overall quantity of items in group G to be offered for sale in the auction or, in the case of an auction to buy (i.e., a procurement auction or a "reverse auction"), to the overall quantity of items in group G to be bought in the auction. The vector of available quantities for all groups will be denoted  $(\overline{Q}^1, \dots, \overline{Q}^h)$  Optionally, the available quantities may be allowed to depend on the prices, or otherwise be contingent on the progress of the auction.

The <u>current prices</u> comprise a vector,  $(P_1, ..., P_m)$ , whose components represent the prices for the *m* respective types of items.

The <u>current bid of bidder i</u> comprises a vector,  $(Q_1^i, ..., Q_m^i)$ , whose components represent the quantities that bidder i is willing to buy (in the case of an auction to sell) or to sell (in the case of an auction to buy) at the current prices for the m respective types of items.

The <u>current bids</u> comprise the collection of vectors,  $\{Q_1^i, \dots, Q_m^i\}_{i=1}^n$ , consisting of the current bid of bidder i for every bidder  $(i = 1, \dots, n)$  in the auction.

The <u>bidding history</u> comprises the current prices and the current bids associated with the present time and all earlier times in the current auction.

A <u>clock auction</u> is a dynamic auction procedure whereby: the auctioneer announces the current prices to bidders; the bidders respond with current bids; the auctioneer determines whether the auction should continue based on the bidding history; the auctioneer updates the current prices based on the bidding history and the process repeats, if it is determined that the auction should continue; and the auctioneer allocates the items among the bidders and assesses payments among the bidders based on the bidding history, if it is determined that the auction should not continue.

Observe that a "clock auction" differs from a standard ascending-bid electronic auction in the following important sense. In standard ascending-bid electronic auctions—such as in the Federal Communications Commission auctions for radio communications spectrum or in eBay auctions—the bidders name prices (and, perhaps also, quantities) that they propose to pay for the items being auctioned, in an iterative process. In a standard clock auction, the auctioneer sets the pace for price increases, and bidders respond only with quantities in an iterative process

Figure 4 is a diagram depicting an exemplary hybrid auction, which combines a clock auction phase with a proxy auction phase. The process starts with step 1002, in which memory locations of a computer are initialized. In one preferred embodiment, the appropriate memory locations of the bidding information processor (auction computer) are initialized with information such as the types of items in the auction, the available quantity of each type of item in the auction, an initial price parameter, an auction timetable, a list of bidder ID's, and a list of passwords. In step 1004, the computer implements the clock auction phase. The clock auction phase of the process is shown in detail in Figure 8. After the clock auction phase concludes (step 118 of Figure 8), the computer proceeds to step 1006, in which it carries forward all of the bids of each bidder from the clock auction phase to the proxy auction phase. In the proxy auction phase, these carried-forward bids will be treated in exactly the same manner as bids that are submitted by the respective bidders' proxy agents. In step 1008, the computer implements the proxy auction phase. The proxy auction phase of the process is shown in detail in Figure 5b. After the proxy auction phase concludes (step 140 of Figure 5b), the computer proceeds to step 1010, in which computer outputs a final message which includes the outcome of the proxy auction phase. In many preferred embodiments, the outcome of the proxy auction phase also serves as the outcome of the overall hybrid auction. The process then concludes.

In some preferred embodiments of the invention, a revealed-preference activity rule (also known as a "revealed-preference-based constraint") is imposed on bidders in both the clock auction phase and in the proxy auction phase. For a longer discussion of revealed preference, see the nearly last sections of "System and Method for a Dynamic Auction with Package Bidding," International Patent Application No. US01/43838.

Here is how they are imposed, in one preferred embodiment. Suppose that there are m types of items (1, ..., m) being auctioned, and let s and t be two times in the auction (s < t). Let  $p^s = (p_1^s, ..., p_m^s)$  be the prices (per unit) for the respective types of items at time s, and let  $p^t = (p_1^t, ..., p_m^t)$  be the prices (per unit) for the respective types of items at time t. Further, for a given bidder, let  $x^s = (x_1^s, ..., x_m^s)$  denote the quantities of the respective types of items demanded by the bidder at time s, and let  $x^t = (x_1^t, ..., x_m^t)$  denote the quantities of the respective types of items demanded by the bidder at time t.

The <u>revealed-preference activity rule</u> accepts the bid  $x' \equiv (x_1', ..., x_m')$  in the clock auction phase only if the following inequality holds:

$$(p'-p')\cdot(x'-x')\leq 0$$
, for all  $s < t$ .

The <u>relaxed revealed-preference activity rule</u> accepts the proxy bid  $x^t \equiv (x_1^t, ..., x_m^t)$  in the proxy auction phase only if the following inequality holds:

$$(p'-p')\cdot(x'-\alpha x')\leq 0$$
, for all times s in the clock auction.

The value  $\alpha \ge 1$ . If  $\alpha > 1$ , this means that the revealed-preference activity rule is relaxed in the proxy auction phase. Relaxing the activity rule is useful for preventing collusion in the auction.

Figure 5a is a high-level depiction of the architecture of an exemplary auction system in which bidding is intermediated by proxy agents, and in which changes to the instructions of proxy agents may be allowed or not allowed, in accordance with an embodiment of the present invention. In the exemplary graphical depiction of Figure 5a, the computer system consists of a server and multiple user computers or terminals. User 30 (the auctioneer) communicates with server 10 (the main auction computer) over a network 40. Users 20a-n (the bidders) also communicate with server 10 over a network 40, but all communications from the respective bidders to the auction process are intermediated through the corresponding proxy agents 50a-n. The proxy agents 50a-n are subsystems of the computer system, and they may physically reside

on the bidder computers or terminals 20a-n, the server or auction computer 10, or any other computer.

In Figure 5a, bidders a-n participate in the auction by entering flexible bid information or making changes in their flexible bid information at their bidder computers or BT's (20a-n). The bidders can enter or change their flexible bid information at times when the auction system is set to allow changes in the flexible bid information of the respective bidders. The actual bidding on behalf of the respective bidders is performed by the proxy agents 50a-n acting on behalf of the respective bidders. Based on the respective bidder's flexible bid information, the proxy agent may compute a bid and submit it in the auction process by transmitting it via a network interface. Meanwhile, the server 10 or auctioneer computer or AT 30 may receive submitted bids, process submitted bids, and update the auction state. This is described in greater detail elsewhere in this application. The server 10 or auctioneer computer or AT 30 may also change the setting of the auction system so as to allow or to not allow bidders to make changes to their flexible bid information. One exemplary way in which this may be done is that the server 10 will compute, according to a predetermined rule, whether flexible bid information changes should be allowed and will send out data to the proxy agents 50a-n, the bidder computers or BT's 20a-n and the auctioneer computer or AT 30 indicating whether flexible bid information changes are allowed. The proxy agents or bidder computers carry out the server's instructions on whether flexible bid information changes are allowed. Meanwhile, the auctioneer has final authority over whether flexible bid information changes are allowed, and can override the server's determination in this regard, if desired.

The "server" (or auction computer) typically has a central role, especially with regard to communications. In some preferred embodiments, the server also does all of the computations and stores all of the data. In some embodiments the "auctioneer" is a live person who sits down at the auctioneer terminal, logs in, and makes decisions which affect the conduct of the auction. Decisions that the auctioneer makes include initialization decisions necessary to initialize an auction such as setting the size of bid increments that will be used and setting the round schedules. Other decisions include determining the "final call" and calling the end of the auction (both typically based on computations and a recommendation by the server). Finally the auctioneer can make decisions in exceptional circumstances such as sending out messages to

bidders and placing bids on behalf of bidders whose Internet connections have failed. Thus aside from the initialization decisions and exceptional events, the auctioneer's decisions can be no more than merely confirming recommendations of other entities. Consequently, in many embodiments, the auctions may be completely automatic, i.e., with no need for human intervention by an auctioneer.

#### Flow Diagram of Augmented Dynamic Package-Bidding Auction Process

In dynamic package-bidding auction processes in the prior art, a bid comprises a package of items and an associated price for the package. That is, bidders merely submit bids comprising pairs, (S, P), where  $S \subset \Omega$  is a subset of the set of all items being auctioned and P is a price at which the bidder is offering to transact for the subset S. There is no scope for bidders to include other information, beyond S and P, in their bids. Furthermore, the provisional revenues are computed simply by optimizing an objective function comprising the sum of the prices in the selected bids, subject to a selection constraint that the bids are compatible (e.g., at most one bid is selected for each item being auctioned). There is no scope for the auction computer to include, in the objective function being optimized or in the selection constraint being applied, the other information that might be explicitly included in bids. Nor is there scope for the auction computer to include, in the objective function being optimized or in the selection constraint being applied, bidder-specific attributes that might be implicitly included in bids (via the identity of the qualified bidder submitting a given bid).

The limitations in the prior art, as summarized in the previous paragraph, limit the applicability and usefulness of dynamic package-bidding auction processes. Conversely, an "augmented dynamic package-bidding auction process," in which any of the limitations summarized in the previous paragraph (or combinations thereof) are eliminated, offers a myriad of new and useful applications. An <u>augmented dynamic package-bidding auction process</u> is thus defined to be any dynamic auction in which package bids are allowed, which includes one or more of the following features: bidders may include other information, beyond a package of items and an associated price for the package, in their bids; the auction computer may include, in the objective function being optimized or in the selection constraint being applied, the other information that might be explicitly included in bids; and the auction computer may include, in

the objective function being optimized or in the selection constraint being applied, bidderspecific attributes that might be implicitly included in bids (via the identity of the qualified bidder submitting a given bid). An augmented dynamic package-bidding auction process may yield efficient outcomes, taking the other information and bidder-specific attributes into account.

The following are some examples of the "other information" that might explicitly be included in bids, to useful effect:

- The terms of payment (e.g., cash-on-delivery versus payment in 30 days)
- The use to which the auctioned items will be put, in a government auction
- The quality of the items being provided, in a procurement auction
- The delivery times of the items being provided, in a procurement auction

The following are some examples of the "bidder-specific information" that might implicitly be taken to be included in bids of qualified bidders, to useful effect:

- The length of time that the bidder has been in business
- The credit-rating of the bidder
- The location of the bidder
- The status of the bidder as a minority-owned business or a small business
- The status of the bidder as a domestic or foreign firm

The following are some examples of how the "other information" or "bidder-specific information" might be included, in the objective function being optimized, to useful effect:

- A higher rating may be assigned to higher-quality items being provided
- A higher rating may be assigned to a selection of bids which includes at least two
  provisional winners that are minority-owned businesses or small businesses
- A higher rating may be assigned to a selection of bids for which at least 50% of each type of good is available for delivery within one week

The following are some examples of how the "other information" or "bidder-specific information" might be included, in the selection constraint, to useful effect:

 A selection constraint may be applied that at least one-third of each type of good be provided by an alternate supplier (second-sourcing)

- A selection constraint may be applied requiring that at least two provisional winners be minority-owned businesses or small businesses
- A selection constraint may be applied requiring that at least 50% of each type of good be available for delivery within one week

An augmented dynamic package-bidding auction process may be implemented on a computer in a system with mandatory proxy bidding, according to Figure 5b.

#### Flow Diagram of Proxy Auction Phase

Figure 5b is a flow diagram of the later phase of an auction in accordance with an embodiment of the present invention: a proxy auction, in which it is mandatory that bidding be intermediated by proxy agents. The process starts with step 122, in which memory locations of a computer are initialized. In one preferred embodiment, the appropriate memory locations of the auction server are initialized with information such as the items in the auction, the auction schedule, the minimum opening bids or reserve prices, a list of bidder ID's, a list of passwords, a list of constraints on bids, and a list of the bids of each bidder from the clock auction phase. These were carried forward in step 1006 of Figure 4, for use in the proxy auction phase. In step 124, a computer outputs the current auction information (if any) available to bidders, possibly including, for example, the minimum opening bids or current high bids, and whether one or more bidders have been given a "last call" for making changes to their flexible bid information. In one preferred embodiment, the auction server outputs the auction information through its network interface and transmits it via the network. The user computers or terminals then receive the auction information through their network interfaces and display the information to bidders and the auctioneer through their user interfaces. In step 126, changes to the flexible bid information for given bidders are entered into computer databases or memory, provided that changes are permitted for the respective bidders (and provided that the bidders wish to make changes to their flexible bid information). This step is illustrated in greater detail in Figures 6a and 6b. In one preferred embodiment, a bidder inputs his (or her) flexible bid information through the user interface of the bidder computer or terminal, which then (if necessary) outputs the auction information through its network interface and transmits it via the network. The proxy agent corresponding to that bidder (if located on another computer) then receives the flexible bid

information through its network interface for use in the next step. In step 128, the proxy agents compute new bids, based on the flexible bid information and the current auction information, to submit on behalf of their respective bidders, and the proxy agents submit new bids (if any) in the auction process on behalf of their respective bidders. This step is illustrated in greater detail in Figures 7a and 7b. In many preferred embodiments, bids comprise pairs (S,P), where  $S \subset \Omega$  is a subset of the set of all items being auctioned and P is a price at which the bidder is offering to transact for the subset S. Stated differently, a bid comprises a package of items and an associated price for the package. As already defined above, such a bid comprising a pair, (S, P), is defined to be a "package bid." In one preferred embodiment, the proxy agents reside on the auction server, so that they can submit new package bids without making use of the network. In a second preferred embodiment, the proxy agents reside on the bidder computers or terminals, in which case the bidder computers or terminals output the submitted new bids through their network interfaces and transmit them via the network. The auction server then receives the submitted new bids through its network interface for use in the next step. In step 130, a computer applies constraints, if any, to the new bids submitted by the proxy agents, and enters only those bids that satisfy said constraints. In one preferred embodiment, the constraints are applied at the auction server, although they may also easily be applied at the bidder computers or terminals, or at other computers.

In step 132, a computer calculates the provisionally-winning bids and provisional revenues, based on the new bids entered and the previous bids that remain "in effect" (i.e., the previous bids that remain active, or remain subject to being selected as winning bids). In one preferred embodiment, the previous bids that remain "in effect" include all of the bids of each bidder from the clock auction phase. In this preferred embodiment, all bids take the form of package bids, all bids that are entered at any time during the auction remain in effect for the duration of the auction, and all bids that are entered on behalf of a given bidder are treated as being mutually exclusive. Therefore, in this preferred embodiment, a computer (which may be the auction server or some other computer) calculates a solution to the following problem of optimizing bid revenues over compatible bids:

Find an n-tuple,  $\{(S_1,P_1),...,(S_n,P_n)\}$ , of bids, one from each bidder i (i = 1,...,n), which maximizes the sum  $P_1 + ... + P_n$ , subject to the constraint that the  $S_i$  are

disjoint subsets of  $\Omega$ . Stated differently, for every i (i = 1,...,n) and for every  $j \neq i$  (j = 1,...,n), it is required that  $(S_i,P_i)$  be a new or previous bid entered by or on behalf of bidder i,  $(S_j,P_j)$  be a new or previous bid entered by or on behalf of bidder j, and  $S_i \cap S_j = \emptyset$ , i.e. no item of set  $S_i$  is a member of the set  $S_j$  if  $i \neq j$ .

In performing the above calculation, the computer may take as implicit the existence of a zero bid, i.e. the pair  $(\emptyset,0)$ , associated with each bidder. The calculated n-tuple,  $\{(S_1,P_1),...,(S_n,P_n)\}$ , of bids solving the above optimization problem is defined to be the provisionally-winning bids; and the calculated sum  $P_1 + ... + P_n$  is defined to be the provisional revenues. However, in other preferred embodiments: (a) only some of the bids that were previously entered into the auction remain in effect for subsequent calculations of the provisionally-winning bids; (b) not all bids that are entered on behalf of a given bidder are treated as being mutually exclusive, so that the optimization problem may allow two or more bids by a single bidder to be selected; and (c) the auction may be an auction to buy, a procurement auction or a reverse auction (rather than an auction to sell), so that the optimization problem for calculating provisionally-winning bids may involve the minimization of payments associated with selected bids, or some other optimization problem, rather than the maximization problem stated above. Also, in many preferred embodiments, a computer stores the calculated provisionally-winning bids and provisional revenues in memory or on a data storage device for future use. In step 134, a computer determines whether the auction should continue. One exemplary way to perform step 134 is for the auction server to compare the current provisional revenues with a function of the provisional revenues obtained in previous iteration(s) of the loop, and to continue the auction if and only if the current provisional revenues exceed the function of the provisional revenues obtained in previous iteration(s). However, this particular stopping rule is only exemplary, and many other embodiments are also possible: for example, the rule applied may be different, it may be performed on a different computer, and the computer may only produce a recommendation of . stopping the auction which is then transmitted to the auctioneer computer or terminal for final approval.

If the auction should continue, the process goes to step 136, where it is determined whether one or more bidders should be given a "last call" to change their flexible bid information. The auction server recommends a decision on whether bidders should be given a

"last call" and transmits this recommendation via the network to the auctioneer computer or terminal. The auctioneer computer or terminal then receives the recommendation through its network interface and displays it to the auctioneer through its user interface. The auctioneer either approves or modifies the recommendation through the user interface of the auctioneer terminal, which then outputs the final decision through its network interface and transmits it via the network. The auction server then receives the final decision through its network interface for use in subsequent steps. The process then goes to step 138, in which the state of the auction system and the current auction information are updated. In one preferred embodiment, the auction server: adds the newly-submitted bids that were entered in step 130 to the list of previous bids that remain in effect; replaces the previous provisionally-winning bids with the provisionally-winning bids that were calculated in the most recent execution of step 132; and replaces the previous provisional revenues with the provisional revenues that were calculated in the most recent execution of step 132. In a second preferred embodiment, the auction server additionally deletes some of the bids from the list of previous bids that remain in effect, in order to reduce the size of the problem that the computer will face at the next iteration of step 132. The process then loops to step 124.

If the auction should not continue, the process goes to step 140, in which a computer outputs a final message, including the allocation of items among bidders and the payments of the bidders. In one preferred embodiment, the auction server recalls its calculation of the provisionally-winning bids at the most recent execution of step 132 and outputs this in a final message as the determined allocation of items among bidders and the payments of the bidders. The auction server outputs this final message through its network interface and transmits it via the network. The bidder and auctioneer computers or terminals then receive the final message through their network interfaces and display the information to bidders and the auctioneer through their user interfaces. The process then ends.

## Detail Elements Concerning Bidders Changing Flexible Bid Information

Figure 6a is a flow diagram illustrating an exemplary process by which a bidder may enter flexible bid information into a computer database or change existing flexible bid

information. Thus, Figure 6a illustrates, in greater detail, step 126 of Figure 5b. The flexible bid information of Figure 6a concerns the bidder's valuations for various items in the auction.

The process starts with step 202, in which bidder i selects a subset  $S \subset \Omega$  of the set of all items being auctioned. In one preferred embodiment, bidder i enters his (or her) selection of subset S through the user interface of his bidder computer or terminal, which then (if necessary) outputs his selection through its network interface and transmits it via the network. The proxy agent of bidder i (if located on another computer) then receives the selection of subset S through its network interface for use in the next step. In step 204, the proxy agent of bidder i recalls the current valuation, v<sub>i</sub>(S) (if any), currently associated with subset S. In one preferred embodiment, the proxy agent of bidder i queries its database to obtain the current valuation v<sub>i</sub>(S), and then (if necessary) outputs the current valuation v<sub>i</sub>(S) through its network interface and transmits it via the network. The bidder computer or terminal of bidder i then receives the current valuation v<sub>i</sub>(S) through its network interface (if the proxy agent is located on a different computer) and displays it on its user interface. In step 206, bidder i inputs a new valuation to be associated with subset S (or cancels input of a new valuation for subset S). As before, in one preferred embodiment, bidder i enters the new valuation through the user interface of his bidder computer or terminal, which then (if necessary) outputs the new valuation through its network interface and transmits it via the network. The proxy agent of bidder i (if located on another computer) then receives the new valuation through its network interface for use in the following steps. In step 208, a computer determines whether changes to the flexible bid information of bidder i are allowed. In one preferred embodiment, the proxy agent of bidder i merely refers to a variable located in the memory of the same computer on which the proxy agent of bidder i resides. If this variable equals one, then changes to the flexible bid information of bidder i are allowed; and if this variable equals zero, then changes to the flexible bid information of bidder i are not allowed. If changes to the flexible bid information of bidder i are allowed, the process continues with step 210, where the proxy agent of bidder i sets v<sub>i</sub>(S) equal to the new valuation that was inputted for subset S in step 206. If changes to the flexible bid information are not allowed, or following step 210, the process goes to step 212, in which it is determined whether bidder i wishes to continue changing his flexible bid information. In one preferred embodiment, the bidder computer or terminal of bidder i displays this as a question through its user interface, bidder i responds to this

question through its user interface, and bidder i's response is transmitted to any other components of the system requiring his response through the network. If bidder i wishes to continue changing his flexible bid information, the process loops back to step 202; otherwise, the process ends.

Figure 6b is a flow diagram illustrating another exemplary process by which a bidder may enter flexible bid information into a computer database or change his existing flexible bid information. Thus, Figure 6b illustrates, in greater detail, step 126 of Figure 5b. The flexible bid information of Figure 6b may concern the bidder's valuations for various items in the auction or may concern a budget limit or parameter.

The process starts with step 252, in which bidder i indicates whether he wishes to change his valuation of a subset, or whether he wishes to change his budget limit or parameter. If bidder i wishes to change his flexible bid information for a valuation of a subset, then the process goes to step 254, in which bidder i selects a subset  $S \subset \Omega$  of the set of all items being auctioned. In one preferred embodiment, bidder i enters his selection of subset S through the user interface of his bidder computer or terminal, which then (if necessary) outputs his selection through its network interface and transmits it via the network. The proxy agent of bidder i (if located on another computer) then receives the selection of subset S through its network interface for use in the next step. In step 256, the proxy agent of bidder i recalls the current valuation, v<sub>i</sub>(S) (if any), currently associated with subset S. In one preferred embodiment, the proxy agent of bidder i queries its database to obtain the current valuation  $v_i(S)$ , and then (if necessary) outputs the current valuation v<sub>i</sub>(S) through its network interface and transmits it via the network. The bidder computer or terminal of bidder i then receives the current valuation v<sub>i</sub>(S) through its network interface (if the proxy agent is located on a different computer) and displays it on its user interface. In step 258, bidder i inputs a new valuation to be associated with subset S (or cancels input of a new valuation for subset S). As before, in one preferred embodiment, bidder i enters the new valuation through the user interface of his bidder computer or terminal, which then (if necessary) outputs the new valuation through its network interface and transmits it via the network. The proxy agent of bidder i (if located on another computer) then receives the new valuation through its network interface for use in the following steps. In step 260, a computer determines whether changes to the flexible bid information of bidder i are allowed. In one

preferred embodiment, the proxy agent of bidder i merely refers to a variable located in the memory of the same computer on which the proxy agent of bidder i resides. If this variable equals one, then changes to the flexible bid information of bidder i are allowed; and if this variable equals zero, then changes to the flexible bid information of bidder i are not allowed. If changes to the flexible bid information of bidder i are allowed, the process continues with step 262, where the proxy agent of bidder i sets  $v_i(S)$  equal to the new valuation that was inputted for subset S in step 258. If changes to the flexible bid information are not allowed, or following step 262, the process goes to step 272.

If bidder i wishes to change his flexible bid information for a budget limit or parameter, then the process goes to step 264, in which the proxy agent of bidder i recalls the current budget limit or parameter. In one preferred embodiment, the proxy agent of bidder i queries its database to obtain the current budget limit or parameter, and then (if necessary) outputs the current budget limit or parameter through its network interface and transmits it via the network. The bidder computer or terminal of bidder i then receives the current budget limit or parameter through its network interface (if the proxy agent is located on a different computer) and displays it on its user interface. In step 266, bidder i inputs a new budget limit or parameter (or cancels input of a new budget limit or parameter). In one preferred embodiment, bidder i enters the new budget limit or parameter through the user interface of his bidder computer or terminal, which then (if necessary) outputs the new budget limit or parameter through its network interface and transmits it via the network. The proxy agent of bidder i (if located on another computer) then receives the new budget limit or parameter through its network interface for use in the following steps. In step 268, a computer determines whether changes to the flexible bid information of bidder i are allowed. In one preferred embodiment, the proxy agent of bidder i merely refers to a variable located in the memory of the same computer on which the proxy agent of bidder i resides. If this variable equals one, then changes to the flexible bid information of bidder i are allowed; and if this variable equals zero, then changes to the flexible bid information of bidder i are not allowed. If changes to the flexible bid information of bidder i are allowed, the process continues with step 270, where the proxy agent of bidder i sets the budget limit or parameter equal to the new value that was inputted in step 266. If changes to the flexible bid information are not allowed, or following step 270, the process goes to step 272.

In step 272, it is determined whether bidder i wishes to continue changing his flexible bid information. In one preferred embodiment, the bidder computer or terminal of bidder i displays this as a question through its user interface, bidder i responds to this question through its user interface, and bidder i's response is transmitted to any other components of the system requiring his response through the network. If bidder i wishes to continue changing his flexible bid information, the process loops back to step 252; otherwise, the process ends.

#### Detail Elements Concerning Bid Submission by Proxy Agents

Figure 7a is a flow diagram illustrating an exemplary process by which a proxy agent may submit new bids based on a bidder's flexible bid information and the current auction information. Thus, Figure 7a illustrates, in greater detail, step 128 of Figure 5b. The flexible bid information of Figure 7a concerns the bidder's valuations for various items in the auction.

The process starts with step 302, in which the proxy agent of bidder i selects an arbitrary subset  $R \subset \Omega$  of the set of all items being auctioned. Subset R is treated as the candidate package on which bidder i is to bid (until a better subset is found). The process goes to step 304, in which the proxy agent of bidder i selects a subset  $S \subset \Omega$  that has not yet been considered. At step 306, the proxy agent recalls the minimum bids, B<sub>i</sub>(R) and B<sub>i</sub>(S), that bidder i is permitted to place on subsets R and S, respectively. In one preferred embodiment, the proxy agent of bidder i queries a database as to the values of B<sub>i</sub>(R) and B<sub>i</sub>(S). (If the proxy agent of bidder i and the database containing the values of B<sub>i</sub>(R) and B<sub>i</sub>(S) are located on different computers, then this communication occurs through the network interfaces of the respective computers and via the network.) In another preferred embodiment, the proxy agent of bidder i outputs the query through the network interface of the computer on which it is located and transmits the query via the network. The auction server then receives the query through its network interface (if located on another computer). The auction server then determines the values of  $B_i(R)$  and  $B_i(S)$  by calculations on data in the state of the auction system. The auction server then outputs the values of B<sub>i</sub>(R) and B<sub>i</sub>(S) through its network interface and transmits them via the network (if necessary). The proxy agent of bidder i then receives the values of B<sub>i</sub>(R) and B<sub>i</sub>(S) through the network interface of the computer on which it is located (if the proxy agent is located on a different computer), making it available for later steps. One exemplary calculation for

determining the values of  $B_i(R)$  and  $B_i(S)$  is for the auction server to take the previous high prices bid for R and S and to multiply each by a positive constant. A second exemplary calculation for determining the value of  $B_i(R)$  is for the auction server to solve the following problem: what is the minimum bid (R, P) that could be submitted by bidder i such that, if provisionally-winning bids were calculated (see step 132 or 164, above) with the extra bid (R, P) included, then (R, P) would be a provisionally-winning bid? (An analogous calculation would then determine  $B_i(S)$ .) The process then goes to step 308, in which a computer determines whether  $v_i(S) - B_i(S) > v_i(R) - B_i(R)$ . In one preferred embodiment, the proxy agent of bidder i merely refers to variables  $v_i(R)$  and  $v_i(S)$ , located in the memory of the same computer on which the proxy agent of bidder i resides, and performs this determination. If  $v_i(S) - B_i(S) > v_i(R) - B_i(R)$ , then the process goes to step 310, where a computer sets R = S (i.e., subset S replaces subset R as the candidate package on which the proxy agent of bidder i is to bid). If  $v_i(S) - B_i(S) \le v_i(R) - B_i(R)$ , or after step 310, the process continues to step 312, in which a computer determines whether all subsets  $S \subset \Omega$  have been considered. If not all subsets  $S \subset \Omega$  have been considered, the process loops back to step 304.

If all subsets  $S \subset \Omega$  have been considered, the process goes to step 314, in which a computer determines whether  $v_i(R) - B_i(R) > 0$ , that is, whether bidder i would receive positive surplus from a winning bid of  $(R, B_i(R))$ . If  $v_i(R) - B_i(R)$  is determined not to be greater than zero, the process jumps to step 320, in which the proxy agent does not place any new bids on behalf of bidder i, and the process ends. If  $v_i(R) - B_i(R)$  is determined to be greater than zero, the process continues to step 316, in which the proxy agent of bidder i determines whether bidder i currently has a provisionally-winning bid on some package A at price  $P_i(A)$ . In one preferred embodiment, the proxy agent of bidder i merely refers to variables, representing the current provisionally-winning bids of bidder i, located in the memory of the same computer on which the proxy agent of bidder i resides, and performs this determination. If bidder i does not currently have a provisionally-winning bid, the process skips to step 322. If bidder i does currently have a provisionally-winning bid on some package A at price  $P_i(A)$ , the process goes to step 318, in which a computer determines whether  $v_i(R) - B_i(R) > v_i(A) - P_i(A)$ , that is, whether bidder i would receive greater positive surplus from a winning bid of  $(R, B_i(R))$  than from a winning bid of  $(A, P_i(A))$ . If  $v_i(R) - B_i(R)$  is determined not to be greater than  $v_i(A) - P_i(A)$ , the process

continues to step 320, in which the proxy agent does not place any new bids on behalf of bidder i, and the process ends. If  $v_i(R) - B_i(R)$  is determined to be greater than  $v_i(A) - P_i(A)$ , the process continues to step 322.

At step 322, the proxy agent submits a new bid on behalf of bidder i for package R at price B<sub>i</sub>(R). In one preferred embodiment, the proxy agent of bidder i outputs the bid (R, B<sub>i</sub>(R)) through the network interface of the computer on which it is located and transmits the submitted bid via the network. The auction server then receives the submitted bid through its network interface (if located on another computer), and utilizes the submitted bid in subsequent steps (for example, step 130 or step 162). After step 322, the process ends.

In another embodiment of the present invention, Figure 7a may be modified so that the proxy agent of bidder i submits two or more new bids, if bidder i would be indifferent among these bids. Step 308 would be expanded so that a computer determines whether  $v_i(S) - B_i(S) = v_i(R) - B_i(R)$ . In that event, step 310 would maintain both R and S as candidate packages on which the proxy agent of bidder i is to bid, and step 322 would have the proxy agent submit bids both of  $(R, B_i(R))$  and  $(S, B_i(S))$ .

In other embodiments of the present invention, Figure 7a is easily modified so that the proxy agent of bidder i bids on behalf of bidder i in a reverse auction or procurement auction. In one such embodiment, Step 306 is modified so that the bids,  $B_i(R)$  and  $B_i(S)$ , are maximum bids that bidder i is permitted to place on subsets R and S, respectively. Step 308 is modified so that a computer determines whether  $B_i(S) - v_i(S) > B_i(R) - v_i(R)$ , since  $B_i(R)$  and  $B_i(S)$  now represent payments that the bidder is willing to accept, while  $v_i(R)$  and  $v_i(S)$  now represent costs of the bidder. Step 314 is modified so that a computer determines whether  $B_i(R) - v_i(R) > 0$ , since this now determines whether bidder i would receive positive surplus from a winning bid of  $(R, B_i(R))$ . Step 318 is modified so that a computer determines whether  $B_i(R) - v_i(R) > B_i(A) - v_i(A)$ , since this now determines whether bidder i would receive greater positive surplus from a winning bid of  $(R, B_i(R))$  than from a winning bid of  $(A, P_i(A))$ .

Figure 7b is a flow diagram illustrating another exemplary process by which a proxy agent may submit new bids based on a bidder's flexible bid information and the current auction information. Thus, Figure 7b illustrates, in greater detail, step 128 of Figure 5b. The flexible bid

information of Figure 7b concerns the bidder's valuations for various items in the auction and a budget limit or parameter.

The process starts with step 352, in which the proxy agent of bidder i selects a subset  $R \subset \Omega$  of the set of all items being auctioned such that the minimum bid,  $B_i(R)$ , that bidder i is permitted to place on subset R is less than or equal to the budget limit or parameter of bidder i. (The proxy agent of bidder i recalls the minimum bid for subset R in the same way as described in step 306 above. If no subset R exists such that the minimum bid, B<sub>i</sub>(R), is within bidder i's budget limit or parameter, then the process jumps all the way to step 372 and does not submit any new bid for bidder i.) Subset R is treated as the candidate package on which bidder i is to bid (until a better subset is found). The process goes to step 354, in which the proxy agent of bidder i selects a subset  $S \subset \Omega$  that has not yet been considered. At step 356, the proxy agent recalls the minimum bids, B<sub>i</sub>(R) and B<sub>i</sub>(S), that bidder i is permitted to place on subsets R and S, respectively. In one preferred embodiment, the proxy agent of bidder i queries a database as to the values of B<sub>i</sub>(R) and B<sub>i</sub>(S). (If the proxy agent of bidder i and the database containing the values of B<sub>i</sub>(R) and B<sub>i</sub>(S) are located on different computers, then this communication occurs through the network interfaces of the respective computers and via the network.) In another preferred embodiment, the proxy agent of bidder i outputs the query through the network interface of the computer on which it is located and transmits the query via the network. The auction server then receives the query through its network interface (if located on another computer). The auction server then determines the values of B<sub>i</sub>(R) and B<sub>i</sub>(S) by calculations on data in the state of the auction system. The auction server then outputs the values of B<sub>i</sub>(R) and B<sub>i</sub>(S) through its network interface and transmits them via the network (if necessary). The proxy agent of bidder i then receives the values of B<sub>i</sub>(R) and B<sub>i</sub>(S) through the network interface of the computer on which it is located (if the proxy agent is located on a different computer), making it available for later steps. One exemplary calculation for determining the values of B<sub>i</sub>(R) and B<sub>i</sub>(S) is for the auction server to take the previous high prices bid for R and S and to multiply each by a positive constant. A second exemplary calculation for determining the value of B<sub>i</sub>(R) is for the auction server to solve the following problem: what is the minimum bid (R, P) that could be submitted by bidder i such that, if provisionally-winning bids were calculated (see step 132 or

164, above) with the extra bid (R, P) included, then (R, P) would be a provisionally-winning bid? (An analogous calculation would then determine B<sub>i</sub>(S).)

The process then goes to step 358, in which a computer determines whether  $B_i(S)$  is less than or equal to the budget limit or parameter of bidder i. If  $B_i(S)$  is greater than bidder i's budget limit or parameter, then the process skips to step 364. If  $B_i(S)$  is less than or equal to bidder i's budget limit or parameter, then the process continues to step 360, where a computer determines whether  $v_i(S) - B_i(S) > v_i(R) - B_i(R)$ . In one preferred embodiment, the proxy agent of bidder i merely refers to variables  $v_i(R)$  and  $v_i(S)$ , located in the memory of the same computer on which the proxy agent of bidder i resides, and performs this determination. If  $v_i(S) - B_i(S) \le v_i(R) - B_i(R)$ , then the process skips to step 364. If  $v_i(S) - B_i(S) > v_i(R) - B_i(R)$ , then the process continues with step 362, where a computer sets R = S (i.e., subset S replaces subset R as the candidate package on which the proxy agent of bidder i is to bid), and then proceeds to step 364. At step 364, a computer determines whether all subsets  $S \subset \Omega$  have been considered. If not all subsets  $S \subset \Omega$  have been considered, the process loops back to step 354.

If all subsets  $S \subset \Omega$  have been considered, the process goes to step 366, in which a computer determines whether  $v_i(R) - B_i(R) > 0$ , that is, whether bidder i would receive positive surplus from a winning bid of  $(R, B_i(R))$ . If  $v_i(R) - B_i(R)$  is determined not to be greater than zero, the process jumps to step 372, in which the proxy agent does not place any new bids on behalf of bidder i, and the process ends. If  $v_i(R) - B_i(R)$  is determined to be greater than zero, the process continues to step 368, in which the proxy agent of bidder i determines whether bidder i currently has a provisionally-winning bid on some package A at price P<sub>i</sub>(A). In one preferred embodiment, the proxy agent of bidder i merely refers to variables, representing the current provisionally-winning bids of bidder i, located in the memory of the same computer on which the proxy agent of bidder i resides, and performs this determination. If bidder i does not currently have a provisionally-winning bid, the process skips to step 374. If bidder i does currently have a provisionally-winning bid on some package A at price  $P_i(A)$ , the process goes to step 370, in which a computer determines whether  $v_i(R) - B_i(R) > v_i(A) - P_i(A)$ , that is, whether bidder i would receive greater positive surplus from a winning bid of (R, B<sub>i</sub>(R)) than from a winning bid of  $(A, P_i(A))$ . If  $v_i(R) - B_i(R)$  is determined not to be greater than  $v_i(A) - P_i(A)$ , the process continues to step 372, in which the proxy agent does not place any new bids on behalf of bidder

i, and the process ends. If  $v_i(R) - B_i(R)$  is determined to be greater than  $v_i(A) - P_i(A)$ , the process continues to step 374.

At step 374, the proxy agent submits a new bid on behalf of bidder i for package R at price  $B_i(R)$ . In one preferred embodiment, the proxy agent of bidder i outputs the bid  $(R, B_i(R))$  through the network interface of the computer on which it is located and transmits the submitted bid via the network. The auction server then receives the submitted bid through its network interface (if located on another computer), and utilizes the submitted bid in subsequent steps (for example, step 130 or step 162). After step 374, the process ends.

In another embodiment of the present invention, Figure 7b may be modified so that the proxy agent of bidder i submits two or more new bids, if bidder i would be indifferent among these bids. Step 360 would be expanded so that a computer determines whether  $v_i(S) - B_i(S) = v_i(R) - B_i(R)$ . In that event, step 362 would maintain both R and S as candidate packages on which the proxy agent of bidder i is to bid, and step 374 would have the proxy agent submit bids both of  $(R, B_i(R))$  and  $(S, B_i(S))$ .

#### Flow Diagram of the Clock Auction Phase

Figure 8 is a flow diagram of an earlier phase of the auction, a clock auction, in accordance with one embodiment of the present invention. The process starts with step 102, in which memory locations of a computer are initialized. In one preferred embodiment, the appropriate memory locations of the bidding information processor (auction computer) are initialized with information such as the types of items in the auction, the available quantity of each type of item in the auction, an initial price parameter, an auction timetable, a list of bidder ID's, and a list of passwords. In step 104, a computer establishes the initial prices  $(P_1, \dots, P_m)$ . The process proceeds to step 106, in which a computer outputs auction information, including the current price vector  $(P_1, \dots, P_m)$ . In one preferred embodiment, the bidding information processor outputs the auction information through its network interface and transmits it via the network. The bidder terminals then receive the auction information through their network interfaces and display the information to bidders through their user interfaces. In step 108, a computer receives bids  $(Q_1^i, \dots, Q_m^i)$  from bidders. In one preferred embodiment, a bidder inputs his bids through the user interface of the bidder terminal, which then outputs the auction

information through its network interface and transmits it via the network. The bidding information processor then receives the bids through its network interface for use in the next step. In step 110, a computer applies constraints, if any, to the received bids, and enters only those bids that satisfy said constraints. This process is illustrated in greater detail in Figures 9a and 9b. In one preferred embodiment, the constraints are applied at the bidding information processor, although they may also easily be applied at the bidder terminals. In step 112, a computer processes the received bids and determines whether the auction should continue. Exemplary processes of step 112 are illustrated in greater detail in Figure 10. In some preferred embodiments, this determination occurs at the bidding information processor.

If the auction should continue, the process goes to step 114, in which a computer establishes an updated price vector  $(P_1, ..., P_m)$ . Then, at step 116, a computer updates other auction information, if any. In one preferred embodiment, the bidding information processor automatically generates a suggested revised price vector, outputs the suggested revised price vector through its network interface, and transmits it via the network. The auctioneer terminal then receives the suggested revised price vector through its network interface and displays it to the auctioneer through its user interface. The auctioneer either approves or modifies the revised price vector through the user interface of the auctioneer terminal, which then outputs the revised price vector through its network interface and transmits it via the network. The bidding information processor then receives the revised price vector through its network interface for use in subsequent steps. The process then loops to step 106.

If the auction should not continue, the process goes to step 118, in which a computer outputs a final message, including the allocation of items among bidders and the payments of the bidders. In one preferred embodiment, the bidding information processor takes the allocation of items among bidders to be their final bids and takes the payment of each bidder i to be the dot product of the final price vector and bidder i's final quantity vector:

$$(P_1, ..., P_m) \cdot (Q_1^i, ..., Q_m^i)$$
.

The bidding information processor outputs the allocation and payment outcome through its network interface and transmits it via the network. The bidder terminals and auctioneer terminal then receive the allocation and payment outcome through their network interfaces and display

the information to bidders and the auctioneer through their user interfaces. The process then ends.

## Elements of the Invention Concerned with Applying Constraints to Bids

Figures 9a and 9b are flow diagrams of two exemplary subprocesses of step 110. The process of Figure 9a begins with step 110a-1, in which a bidder i who has not yet been considered is selected. In step 110a-2, a bid  $\left(Q_k^{i,i}\right)_{k\in G}$  by bidder i which has not yet been considered is selected. The bid  $\left(Q_k^{i,i}\right)_{k\in G}$  is a bid for group G of item types at time t in the auction. In step 110a-3, it is checked whether each quantity  $Q_k^{i,i}$  in the selected bid is a nonnegative integer. If each component of the bid is a nonnegative integer, the process goes to step 110a-4. In step 110a-4, it is checked whether the selected bid is consistent with bidder i's initial eligibility, that is, whether:

$$\sum_{k\in G} Q_k^{i,i} \leq \bar{Q}_G^i,$$

where bidder i's initial eligibility,  $\overline{Q}_G^i$ , for group G may, for example, be determined by the level of financial guarantee posted by bidder i. If the selected bid is consistent with bidder i's initial eligibility, the process goes to step 110a-5, where bidder i's most recent previously-processed bid for group G, denoted  $\left(Q_k^{i,i-1}\right)_{k\in G}$ , is recalled. In step 110a-6, it is checked whether the selected bid is consistent with the auction's activity rule, that is, whether the constraint:

$$\sum_{k\in G} Q_k^{i,t} \leq \sum_{k\in G} Q_k^{i,t-1},$$

is satisfied. If it is, the process continues to step 110a-7, where the selected bid  $(Q_k^{i,i})_{k \in G}$  is entered as a valid bid by bidder i on group G. Optionally, bidder i is sent a message confirming to him that the bid is valid. The process then goes to step 110a-8, where it is determined whether all bids by bidder i have been considered. If not, the process loops back to step 110a-2. If all bids by bidder i have been considered, the process continues to step 110a-9, where it is determined whether all bidders have been considered. If not, the process loops back to step 110a-1. If all bidders have been considered, the process goes to step 112 of Figure 8.

If the selected bid fails any of the checks at steps 110a-3, 110a-4 or 110a-6, the process instead goes to step 110a-10, where a message is outputted to bidder *i* that the selected bid is invalid. The selected bid then is not entered as a valid bid. The process then goes to step 110a-8,

where it is determined whether all bids by bidder *i* have been considered. If not, the process loops back to step 110a-2. If all bids by bidder *i* have been considered, the process continues to step 110a-9, where it is determined whether all bidders have been considered. If not, the process loops back to step 110a-1. If all bidders have been considered, the process goes to step 112 of Figure 8.

The process of Figure 9b begins with step 110b-1, in which a bidder i who has not yet been considered is selected. In step 110b-2, a bid  $\left(Q_k^{i,t}\right)_{k\in G}$  by bidder i which has not yet been considered is selected. The bid  $\left(Q_k^{i,t}\right)_{k\in G}$  is a bid for group G of item types at time t in the auction. In step 110b-3, it is checked whether each quantity  $Q_k^{i,t}$  in the selected bid satisfies the constraint:

$$\sum_{k \in G} C_k^{i,t} Q_k^{i,t} \leq \overline{C}_G^{i,t} ,$$

where  $C_k^{i,i}$  and  $\bar{C}_G^{i,i}$  are arbitrary constants. If the constraint of step 110b-3 is satisfied, the process goes to step 110b-4. In step 110b-4, it is checked whether each quantity  $Q_k^{i,i}$  in the selected bid satisfies the constraint:

$$\sum_{k\in G} C_k^{i,i} Q_k^{i,i} \geq \hat{C}_G^{i,i},$$

where  $C_k^{i,i}$  and  $\hat{C}_G^{i,i}$  are arbitrary constants. If the constraint of step 110b-4 is satisfied, the process goes to step 110b-5, where it is checked whether the selected bid was submitted at a time no earlier than the starting time of the current round. If it was, the process goes to step 110b-6, where it is checked whether the selected bid was submitted at a time no later than the ending time of the current round. If it was, the process continues to step 110b-7, where the selected bid  $(Q_k^{i,i})_{k\in G}$  is entered as a valid bid by bidder i on group G. Optionally, bidder i is sent a message confirming to him that the bid is valid. The process then goes to step 110b-8, where it is determined whether all bids by bidder i have been considered. If not, the process loops back to step 110b-2. If all bids by bidder i have been considered. If not, the process loops back to step 110b-1. If all bidders have been considered, the process goes to step 112 of Figure 8.

If the selected bid fails any of the checks at steps 110b-3, 110b-4, 110b-5 or 110b-6, the process instead goes to step 110b-10, where a message is outputted to bidder *i* that the selected bid is invalid. The selected bid then is not entered as a valid bid. The process then goes to step

110b-8, where it is determined whether all bids by bidder *i* have been considered. If not, the process loops back to step 110b-2. If all bids by bidder *i* have been considered, the process continues to step 110b-9, where it is determined whether all bidders have been considered. If not, the process loops back to step 110b-1. If all bidders have been considered, the process goes to step 112 of Figure 8.

It is important to note that, in many preferred embodiments of the clock auction phase, bidders are allowed full flexibility in making bids which, if accepted, would cause aggregate demand to be less than supply. After each new price vector is announced, bidders can arbitrarily reduce their previous quantities bid. (However, note that the previous bids will be carried forward to the proxy auction phase in many preferred embodiments, so the bids retain meaning.) For example, it might be the case that supply equals demand for a particular item, but a bidder may wish to reduce his demand on that item, as the price of a complementary item has increased. Or it might be the case that, when demand was greater than supply for a particular item, two bidders simultaneously attempted to reduce their demands, sufficiently to now make demand less than supply. It is tempting to refuse to allow the reduction in the first case, or to ration the bidders in the second case, since otherwise the clock auction phase may yield a significant underselling of the items in the auction. However, to refuse the reduction or to ration the bidders may yield an exposure problem for bidders who have complements preferences. Consequently, in many preferred embodiments of the clock auction phase, the full flexibility to reduce arbitrarily reduce bids is allowed. In any event, observe that the clock auction phase does not conclude the auction, and the underselling can be remedied during the proxy auction phase.

## Embodiments of the Invention Concerned with Whether the Clock Phase Should Continue

Figure 10 is a flow diagram of a subprocess of step 112 of Figure 8. It illustrates an exemplary process by which a computer may determine whether the auction should continue. Figure 10 begins with step 112a-1, in which a group G of types of items not yet considered is selected. In step 112a-2, a computer determines whether the excess demand for group G of types of items is within  $\overline{C}^G$  of the available quantity, that is, whether:

$$\left| \bar{Q}^G - \sum_{i=1}^n \sum_{k \in G} Q_k^i \right| \leq \bar{C}^G.$$

The constant,  $\overline{C}^G$ , has the interpretation that this is the tolerance to which the auctioneer is allowing oversell or undersell to occur. If the auctioneer needs to sell exactly the available quantity of the group G of item types, then  $\overline{C}^G = 0$ . If this inequality is not satisfied, then group G of item types has not yet cleared, and so the auction should continue. The process thus jumps immediately to step 114 of Figure 8.

If the inequality of step 112a-2 is satisfied, the process then goes to step 112a-3, where it is determined whether all groups G of types of items have been considered. If not, the process loops back to step 112a-1. However, if all groups G of types of items have already been considered, then it has been found that all groups G of types of items have cleared within a tolerance of  $\overline{C}^G$ , and so the auction should not continue. The process proceeds to step 118 of Figure 8, where the final message is generated.

FIGURE 1

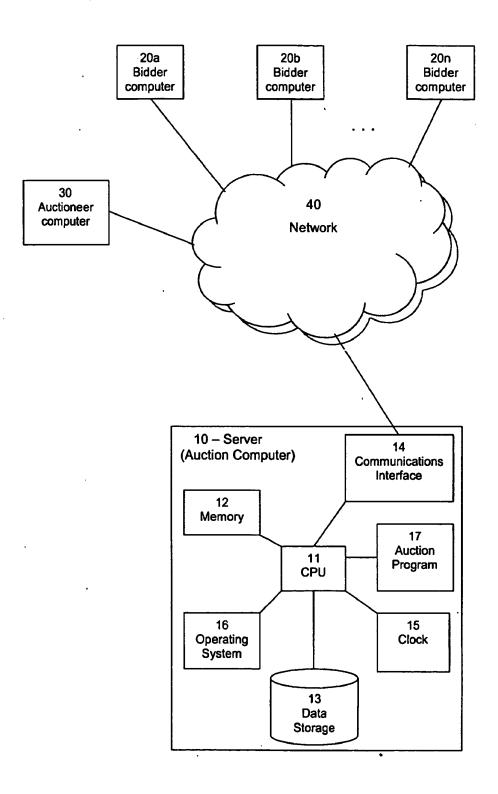


FIGURE 2

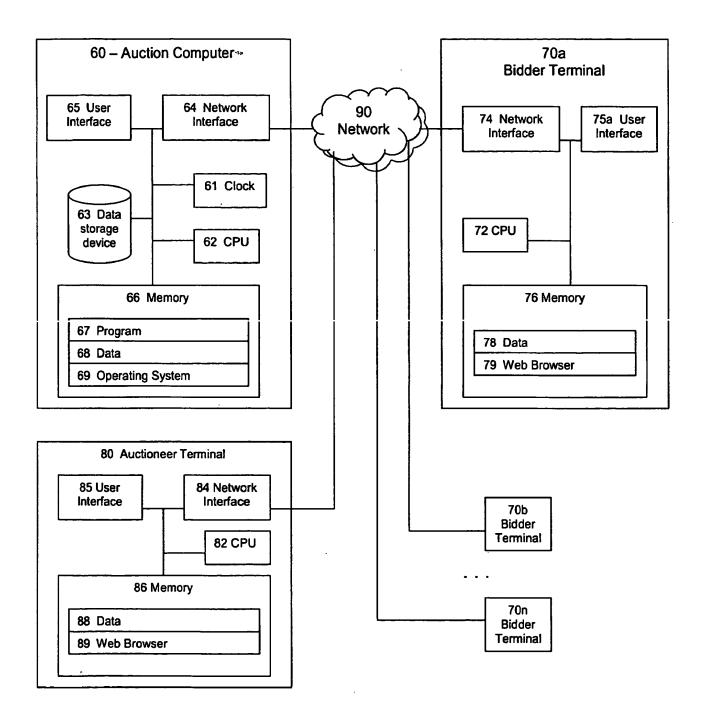


FIGURE 3

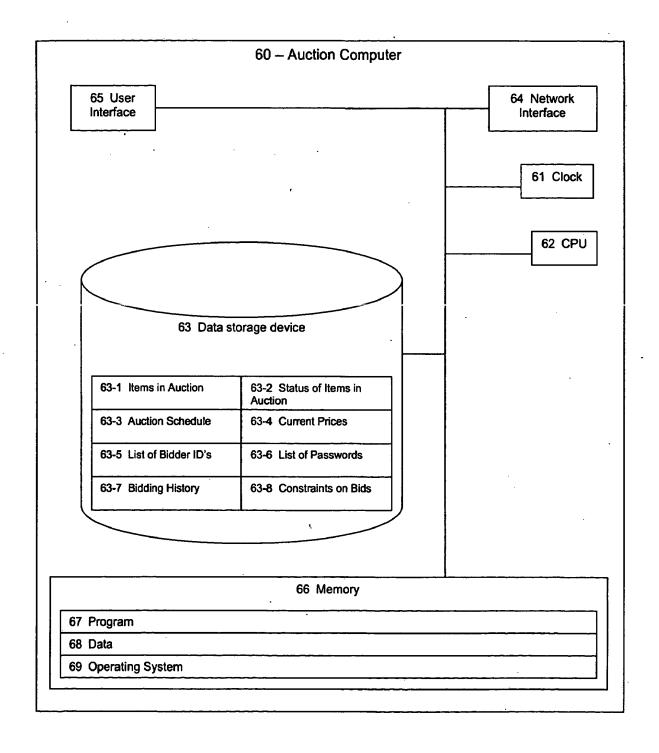
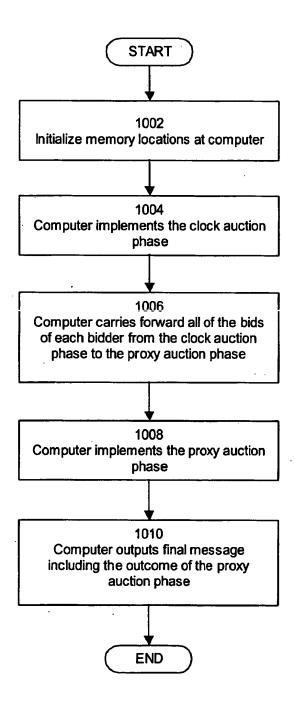
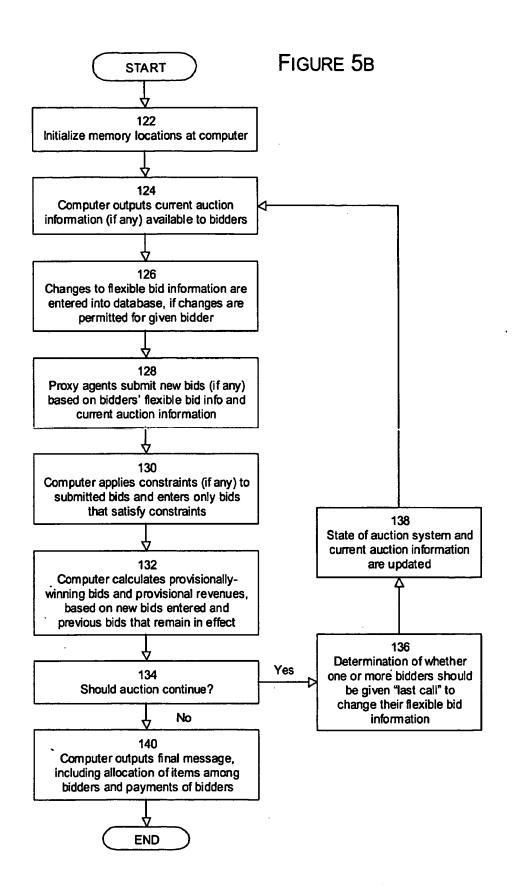
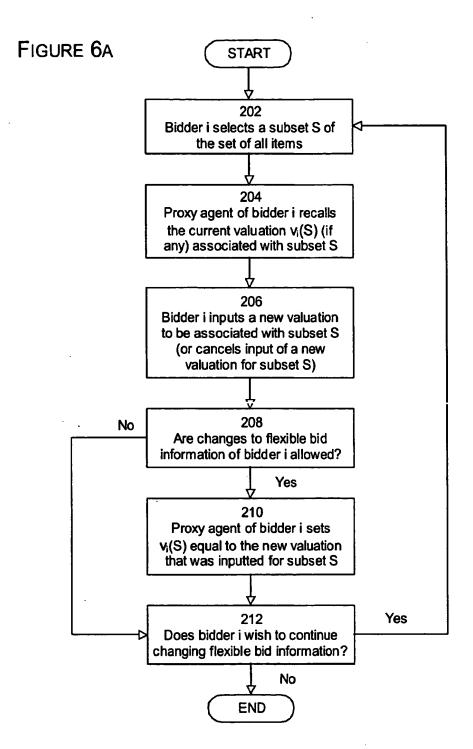
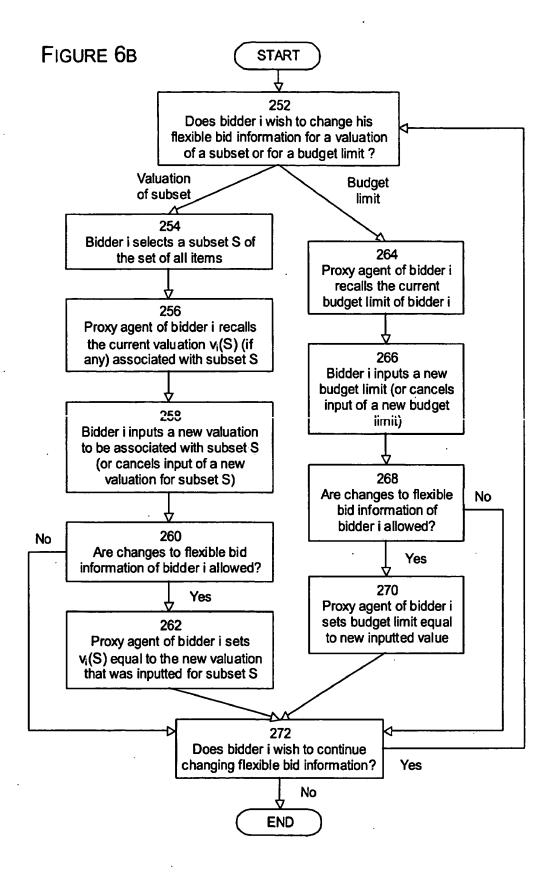


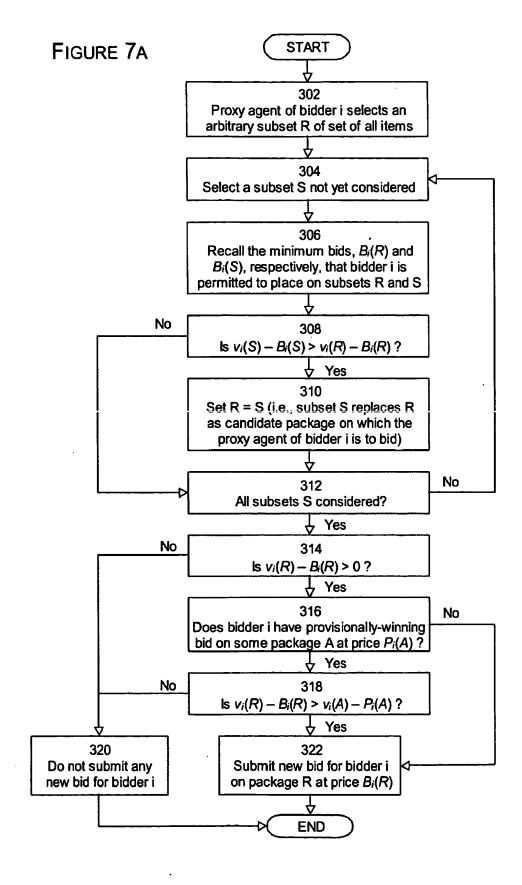
FIGURE 4

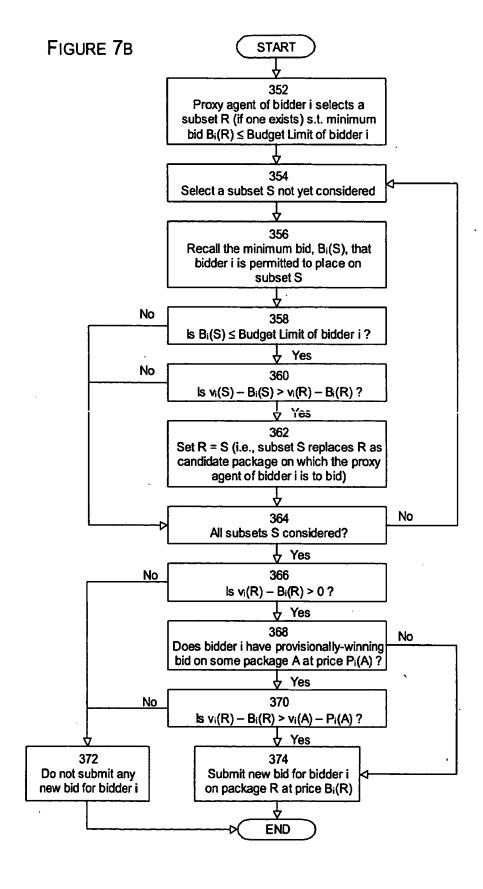


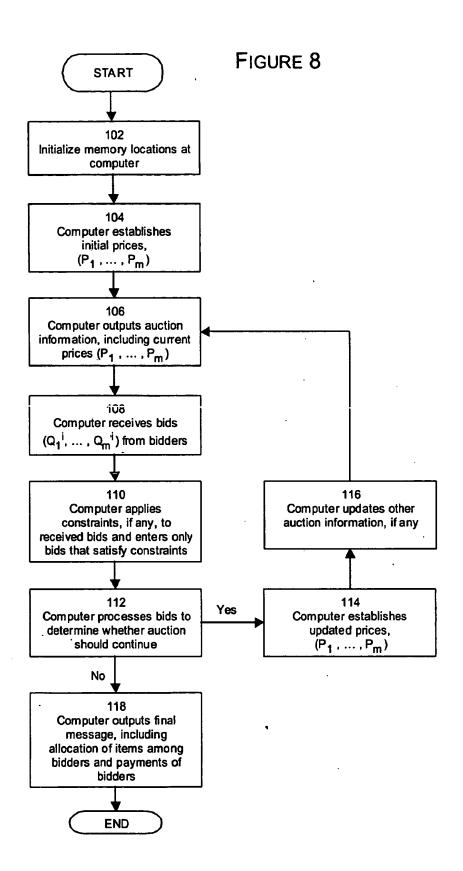


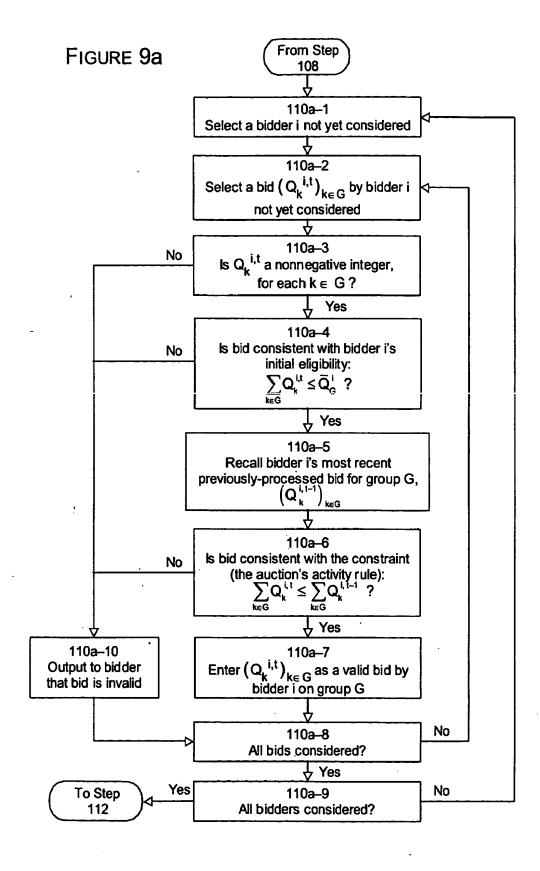












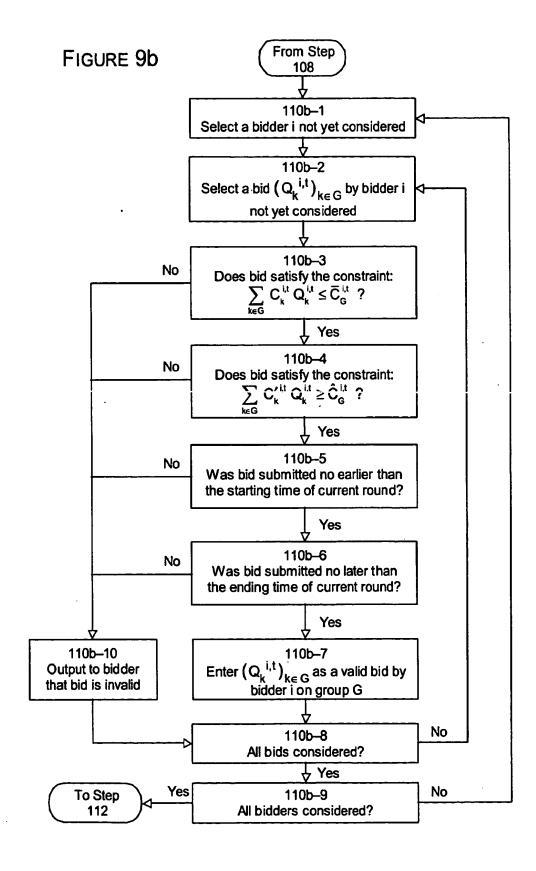
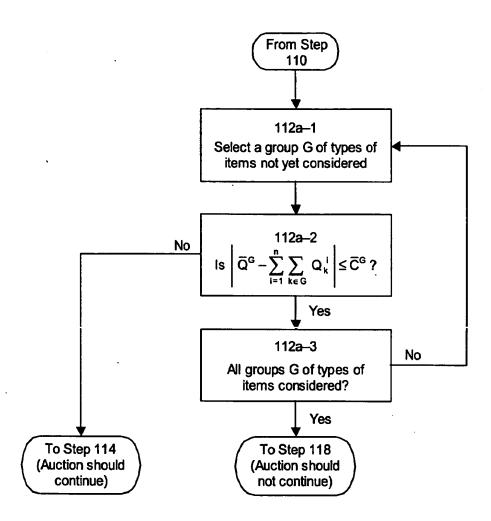


FIGURE 10



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